Performance Evaluation of Frequency Hopped Receiver Oriented Spread Spectrum Packet Radio Networks

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Abstract

In this paper we examine the multiple access capability of frequency hopped, receiver oriented, spread spectrum packet radio networks. The most important indicator of the multiple access capability of a spread spectrum packet radio network is the induced packet error probability. In this work, we first determine the interdependence of byte errors of the desired transmission (i.e., packet) due to the multiple access interference caused by other packets. Then, we propose an exact and two approximation methods for the computation of the packet error probability. Our results indicate that the byte errors exhibit a Markovian structure.

1. Introduction

Frequency hopped spread spectrum (FH–SS) packet radio networks belong to two distinct classes.

Class A**:** Transmitter-oriented FH–SS packet radio networks

Class B**:** Receiver-oriented FH–SS packet radio networks.

In networks of class A, every transmitter in the network is assigned its own frequency hopping pattern. A transmitter sends a packet utilizing its own frequency hopping pattern. As a result, a receiver must monitor all frequency hopping patterns of the transmitters within range. This seems to be unrealistic if the receiver corresponds to a mobile user in a multi-hop packet radio network. On the other hand, in networks of class B, every receiver in the network has its own frequency hopping pattern. A transmitter sends a packet utilizing the frequency hopping pattern of the intended receiver. Consequently, in networks of class B, a receiver has to monitor only its own frequency hopping pattern. This seems to be the only realistic assumption if the receiver is a mobile user.

In this paper, we compute the packet error probability induced in receiver- oriented, frequency hopped spread spectrum packet radio networks. The packet error probability is the best indicator of the multiple access capability of a spread spectrum packet radio network. The interference between packet transmissions in a spread spectrum multiple access environment produces depentent bit or byte errors. Hence, the evaluation of the bit or byte error probability is not sufficient for the computation of the packet error probability. The computation of the packet error probabilty for transmitter oriented, frequency hopped, spread spectrum packet radio networks has been examined before ([1],[2],[3],[4]). It is the first time though, to the best of our knowledge, that the problem of computing packet error probabilities in receiver oriented, frequency hopped, spread spectrum packet radio networks, is formally addressed. In our formulation, the interdependence of bit or byte errors will be taken into account.

2. The Model

In our basic model of a receiver oriented, frequency hopped, spread spectrum packet radio network we make the following assumptions.

A.1) A packet is exactly one codeword from an (M, L) Reed Solomon (RS) code. [M=total number of codeword symbols (bytes); L=number of information symbols (bytes)].

A.2) The dwell interval is equal to the hop interval.

A.3) The dwell (hop) interval contains only one codeword symbol (byte) of the Reed Solomon code.

A.4) The only noise present is due to multiple access interference.

A.5) The channel time is divided into slots and the users in the network initiate their packet transmissions at the beginning of some slot.

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A.6) Every receiver in the network is assigned a random frequency hopping pattern. Different receivers in the network are assigned statistically independent frequency hopping patterns. Every frequency hopping pattern is a sequence of independent random variables. Each random variable is uniformly distributed over a set I of q frequencies (i.e. $I = 1, 2, ..., q$). These patterns are called memoryless hopping patterns.

A.7) A bute hit results in a byte error (we say that a byte of a packet is hit if during its reception by a receiver some other packet occupies the same frequency that this packet occupies).

A.8) The network is a receiver oriented, frequency hopped, spread spectrum packet radio network (this implies that a transmitter sends a packet utilizing the frequency hopping pattern of the intended receiver).

A.9) A receiver locks on to a packet within a slot, if and only if, this packet is the only one utilizing the frequency hopping pattern of the receiver.

In the following section we are going to present three methods for the computation of the packet error probability.

3. Analysis

3.1 Preliminaries

We initially concentrate on the computation of the packet error probability when the maximum propagation delay t in the network is equal to two hops. Then, we will examine the general case. This approach simplifies our presentation considerably.

Let us now denote by $G_i, 1 \leq i \leq K$, the group of packets, which utilize the frequency hopping pattern of receiver $R_i, 1 \leq i \leq K$, within some slot. We assume that the group G_1 consists of a single packet, and we name this packet #1. We also assume that receiver R_1 locks on to packet #1. We denote by $\{f_j^i; 1 \leq j \leq M\}$ the frequency hopping pattern corresponding to receiver R_i. Without loss of generality, we assume that packet #1 arrives at receiver R_1 at the beginning of the slot. The groups of packets, $G_i (2 \leq i \leq K)$, belong to one of the following two types.

1) A group of packets belongs to type 1, if some of the packets in the group arrived within the first hop, and the remaining packets arrived within the second hop after the arrival of packet #1.

2) A group of packets belongs to type 2, if all the packets in the group arrived either in the first hop or in the second hop after the arrival of packet #1.

In Figure 1, we show the frequencies corresponding to bytes $j-1, j$ and $j+1$ of packet #1. In the same Figure, we show the frequencies, utilized by the groups G_2 and G_3 of packets, that affect bytes $j-1, j$ and $j+1$ of packet #1. In Figure 1, we assumed that G_2 is a type 1 group and G_3 is type 2 group. It is easy to see from Figure 1 that the probability of incorrectly decoding packet #1 is not affected if the G_2 group consists of two packets and the G_3 group consists of one packet; one packet in the G_2 group arrives in the first hop and the other packet arrives in the second hop after the arrival of packet #1.

Let us denote by $P_e(K_1, K_2)$ the probability that packet #1 is incorrectly decoded, provided that the multiple access interference is due to K_1 type 1 and K_2 type 2 groups of packets (note that $K_1 + K_2 = K - 1$. In Figure 2, we show the frequencies utilized by bytes $j-1, j$ and $j+1$ of packet #1. In the same Figure, the frequencies, utilized by the bytes of the packet arrivals from the $K-1$ groups that interfere with bytes $j-1, j$ and $j+1$ of packet #1 and affect $P_e(K_1, K_2)$, are also shown.

Our purpose is to compute the packet error probability $P_e(K_1, K_2)$ for various values of K_1 and K_2, such that $K_1 + K_2 = K - 1$ and $K_1 \geq 1$. An exact and two approximation methods are going to be examined.

3.2 The exact Method

Let us denote by $S(m_1, m_2), 1 \leq m_2 \leq M, m_1 \leq m_2$, the number of bytes from byte m_1 to byte m_2 of packet #1 that are in error. Then, we can write

$$P_e(K_1, K_2) = \sum_{i=e+1}^{M} Pr[S(1, M) = i] \qquad (1)$$

where e is the error correction capability of the code ((M, L) RS code).

Let us also define the vectors X_j, Y_j, Z_j, W_j, T_j and the vectors x_j, y_j, z_j, w_j, t_j as follows:

$$X_j = (f_j^2 \dots f_j^{K_1+1}) \ ; \ x_j = (s_j^2 \dots s_j^{K_1+1})$$
$$j = -1, 0, \dots$$

$$Y_j = (f_j^{K_1+2} \dots f_j^K) \ ; \ y_j = (s_j^{K_1+2} \dots s_j^K)$$
$$j = 0, 1, \dots$$

$$Z_j = (X_{j-2} \ \ X_{j-1} \ \ X_j) \ ; \ z_j = (x_{j-2} \ \ x_{j-1} \ \ x_j)$$
$$j = 1, 2, \dots$$

$$W_j = (Y_{j-1} \ \ Y_j) \ ; \ w_j = (y_{j-1} \ \ y_j)$$
$$j = 1, 2, \dots$$

$$T_j = (f_j^1 \;\; Z_j \;\; W_j) \quad ; \quad t_j = (s_j^1 \;\; z_j \;\; w_j)$$
$$j = 1, 2 \ldots$$

where each s_j assumes values from a set I of q frequencies (i.e. $I = \{1, 2, \ldots q\}$). Our goal is to find a way to compute $Pr[S(1, M) = i]$. The first step towards this direction is the following proposition.

Proposition 1: The sequence of random vectors $T_j (j \geq 1)$ is a Markov chain with stationary transition probabilities.

The proof of Proposition 1 is easy and it is ommitted. Let us now consider the conditional probability

$$Pr[S(2, m) = i | (f_1^1 \;\; X_{-1} \;\; X_0 \;\; X_1 \;\; Y_0 \;\; Y_1) = (s_1^1 \;\; x_{-1} \;\; x_0 \;\; x_1 \;\; y_0 \;\; y_1)] \tag{2}$$

with $m \geq 2$, $0 \leq i \leq m - 1$, $s_1^1 \in I$, $x_{-1}, x_0, x_1 \in I^{K_1}$ and $y_0, y_1 \in I^{K_2}$. We can now state one more proposition.

Proposition 2: For every $m \geq 2$ and all i such that $0 \leq i \leq m - 1$ the conditional probability in (2) depends on the following two events.

i) The number of distinct components of the vector x_1.

ii) The number of distinct components of the vector $(x_0 \;\; x_1 \;\; y_1)$

The proof of Proposition 2 is based on Proposition 1. The details are ommitted due to lack of space. Proposition 2 allows us to write the conditional probability in (2) as follows.

$$Pr[S(2, m) = i | l_1, n_1] \tag{3}$$

where l_1 and n_1 are the number of distinct components of the vectors x_1 and $(x_0 \;\; x_1 \;\; y_1)$, respectively. A byproduct of the proof of Proposition 2 (see [5] for more details), is that the conditional probability in (3) satisfies the following recursive expressions.

$$Pr[S(2, m) = i | l_1, n_1] = \sum_l \sum_n C_1(l_1, n_1; l, n) Pr[S(2, m-1) = i - 1 | l, n] + \sum_l \sum_n C_2(l_1, n_1; l, n) Pr[S(2, m-1) = i | l, n] \tag{4}$$

The indices l_1, l in (4) range from 1 to $min(K-1, q)$. Furthermore, the indices n_1 and n range from l_1 to $min(K-1, q-l_1)$ and l to $min(K-1, q-l)$, respectively. The expressions for the coefficients C_1 and C_2 are included in [5]. The importance of formula (4) is that, with known C_1 and C_2, the probabilities $Pr[S(2, m) = i | l_1, n_1]$ can be evaluated recursively.

One more step is needed to compute $Pr[S(1, M) = i]$ for $i = e + 1, e + 2, \ldots M$. We can show that (see [5]).

$$Pr[S(1, M) = i] = \sum_{l_1} \sum_{n_1} C_3(l_1, n_1) \;\; Pr[S(2, M) = i - 1 | l_1, n_1] + \sum_{l_1} \sum_{n_1} C_4(l_1, n_1) \;\; Pr[S(2, M) = i | l_1, n_1] \tag{5}$$

The expressions for the coefficients C_3 and C_4 are also contained in [5]. Formulas (4) and (5) allow us to evaluate $Pr[S(1, M) = i]$ for $i = e + 1, \ldots M$. Then $P_e(K_1, K_2)$ can be computed from (1).

A drawback of the exact method presented above is its computational complexity. The expressions for the coefficients C_1, C_2, C_3 and C_4 are considerably involved, and the computational complexity for their evaluation increases as $K_1 + K_2 = K - 1$ becomes large. Furthermore, the computational complexity associated with the evaluation of the conditional probabilities $Pr[S(2, M) = i | l_1, n_1]$ increases as either M, or $K_1 + K_2 = K - 1$, or both increase (see (4)).

In section 3.3 we are going to examine two approximation methods for the computation of $P_e(K_1, K_2)$. Both of them are less computationally complex than the exact method. In section 3.4, the validity of the approximations will be investigated by presenting numerical results.

3.3 Approximation Methods

Let us denote by $H_j (1 \leq j \leq M)$ random variables such that $H_j = 1$ if the *jth* byte of packet #1 is hit, and $H_j = 0$ otherwise. The computation of the packet error probability, $P_e(K_1, K_2)$, is considerably simplified if we treat the random variables $H_j (1 \leq j \leq M)$ as independent. Then, our first "approximation" to the packet error probability, $P_e(K_1, K_2)$, is given by the following expression.

$$P_{e,i}(K_1, K_2) = \sum_{i=e+1}^{M} \binom{M}{i} p^i (1-p)^{M-i} \tag{6}$$

where p is the byte error probability. It is easy to show that

$$p = 1 - (1 - q^{-1})^{2K + K_1 - 2} \tag{7}$$

Our second "approximation" to the packet error probability can be derived by observing that the random variable H_j depends on the random variables H_{j-1} and H_{j-2}, but it is independent of the random variables $H_{j-3}, H_{j-4}, \ldots, H_1$. This observation leads us to the second "approximation" method, where we treat the random variables $H_j (j \geq 1)$ as a second order Markov chain. That is, we assume that the following equality is true.

$$Pr\{H_j = i_j | \{H_k = i_k; 1 \leq k \leq j-1\}) = Pr(H_j = i_j | H_{j-1} = i_{j-1}, H_{j-2} = i_{j-2}) \quad (8)$$

where $j \geq 4$, $i_k = 0$ or 1 for $1 \leq k \leq j$. Under the assumption that (8) is true, we can compute an "approximation" to the packet error probability $P_e(K_1, K_2)$. We denote this "approximation" $P_{e,M}(K_1, K_2)$. The evaluation of $P_{e,M}(K_1, K_2)$ is made possible due to the fact that (8) allows us to write recursive equations to compute the conditional probabilities

$$Pr[S(3,m) = i | H_1 = i_1, H_2 = i_2] = \quad i_1, i_2 = 0, 1 \; ; \; 0 \leq i \leq m-2 \quad (9)$$

It is worth noting, that the evaluation of $P_{e,M}$ requires the knowledge of the probabilty mass functions of the sequence of the random variables $H_j (j \geq 1)$ up to order three.

3.4 Numerical Results

The approximation methods discussed in the previous section, however reasonable they may seem, are useless if we fail to show their proximity to the exact results presented in section 3.2. We expect the second approximation to be more accurate, because it takes into consideration the dependence of the byte errors. In Table 1, we present the values of $P_e(K_1, K_2), P_{e,i}(K_1, K_2)$ and $P_{e,M}(K_1, K_2)$ for different choices of $K_1 + K_2 = K - 1 (K \geq 2)$, when the (31,15) and the (63,31) RS codes are used for the encoding of the packets. Two observations are pertinent with the results of Table 1.

1) The first approximation method induces a relative error of less than 0.7 % (i.e. $|P_{e,i} - P_e| / P_e < 7 \times 10^{-3}$)

2) The second approximation method gives us a packet error probability $P_{e,M}$, which is equal to the exact packet error probability P_e up to the sixth and sometimes up to the eighth most significant nonzero decimal point.

Similar observations hold for other K_1 and K_2 choices (see [5]). The entries of Table 1 support our previous expectations. The second approximation method is more accurate than the first approximation method. On the other hand, the exact method is more computationally complex than the second approximation method, and the second approximation method is more complex than the first approximation method.

3.5 Extensions

Some of the results presented in sections 3.1-3.4, where $t = 2$ (i.e. maximum propagation delay in the network is equal to two hops), are still applicable in the general case when $t \geq 3$. The details are ommitted due to lack of space (for more information see [5]).

The excellent results of the Markovian assumption, when $t = 2$, and the fact that this assumption takes into consideration (at least partially) the interdependence of byte errors allows us to state the following conjecture.

Conjecture # 1 For networks with maximum propagation delay t (t$\geq$ 2), the byte errors exhibit a t-order Markovian structure.

4. Conclusions

We have examined the problem of computing packet error probabilities in receiver oriented, frequency hopped, spread spectrum packet radio networks. The dependence of byte errors has been completely characterized by proposition 2 and its immediate extensions. An important outcome of our work is the "educated" Conjecture 1, presented in the previous section. Conjecture 1 provides us with a computationally efficient method for the approximate evaluation of the packet error probability induced in the above systems.

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Figure 1

G_2 and G_3 groups of packets interfering with packet #1

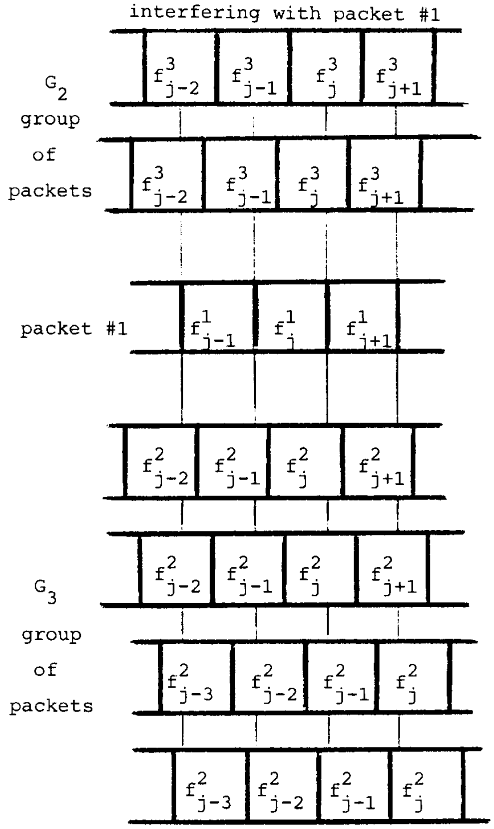

Figure 2

K_1 type 1 and K_2 type 2 groups of packets interfering with packet #1 and affecting $P_e(K_1, K_2)$

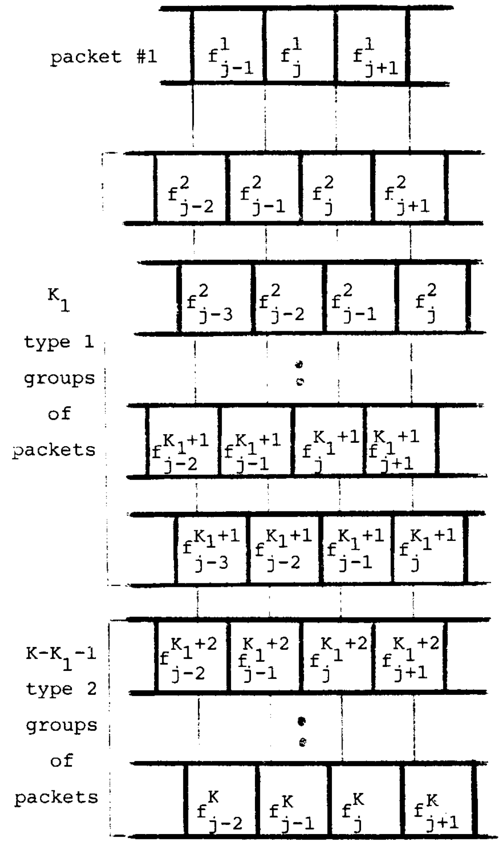

Table 1
Packet Error Probabilities
Maximum propagation delay in the network is equal to two hops
$q = 50$, (31,15) RS code

K_1	K_2	$P_{e,M}$	P_e
1	0	0.51852690D-04	0.51852690D-04
1	1	0.19736752D-02	0.19736752D-02
2	0	0.63506013D-02	0.63505958D-02
1	2	0.15874958D-01	0.15874958D-01
3	0	0.60425529D-01	0.60425489D-01
1	3	0.60360377D-01	0.60360376D-01
K_1	K_2	$P_{e,i}$	$\lvert P_{e,i} - P_e\rvert / P_e$
1	0	0.51665336D-04	0.36%
1	1	0.19679813D-02	0.29%
2	0	0.63257070D-02	0.39%
1	2	0.15842149D-01	0.21%
3	0	0.60278454D-01	0.24%
1	3	0.60278454D-01	0.14%

$q = 50$, (63,31) RS code

K_1	K_2	$P_{e,M}$	P_e
1	0	0.896386D-07	0.896386D-07
1	1	0.67316327D-04	0.67316327D-04
2	0	0.53861051D-03	0.53860964D-03
1	2	0.26749440D-02	0.26749439D-02
3	0	0.25826555D-01	0.25826525D-01
1	3	0.25783565D-01	0.25783565D-01
K_1	K_2	$P_{e,i}$	$\lvert P_{e,i} - P_e\rvert / P_e$
1	0	0.890588D-07	0.67%
1	1	0.66980470D-04	0.5%
2	0	0.53503315D-03	0.66%
1	2	0.26658343D-02	0.34%
3	0	0.25729837D-01	0.37%
1	3	0.25729837D-01	0.21%